

## A PRACTICAL METHOD FOR CALIBRATING A COAXIAL NOISE SOURCE WITH A WAVEGUIDE STANDARD

Yoshihiko Kato and Ichiro Yokoshima

Electrotechnical Laboratory  
1-1-4, Umezono, Sakura-mura, Niihari-gun  
Ibaraki-ken, Japan 305

### Abstract

A practical method for calibrating a coaxial noise source with a waveguide standard has been developed by extending the adaptor-changing method reported before, and a practical equation to give its noise temperature, the measurement procedure and the error analysis are described.

### Introduction

Microwave noise sources are requisite to the quantitative measurements of receiver powers and receiving systems in radio astronomy, earth-space communication, and remote sensing.

As the primary standard, would be preferable thermal noise sources of the waveguide type rather than the coaxial type from the simplicity of their configurations and the ease of the evaluation of their noise temperatures. Meanwhile, those of the coaxial type are widely used as the secondary standard since they are small-sized and light-weighted. Therefore, there will occur the case where the coaxial noise source should be calibrated as to noise temperature with the waveguide standard by placing a coax-waveguide adaptor in front of its output port as shown in Fig.1. In this case, the relationship between the noise temperatures of the coaxial noise source and at the output port of the adaptor,  $T_x$  and  $T_{xw}$ , is expressed as

$$T_{xw} = \alpha_{xw} T_x + (1 - \alpha_{xw}) T_a \quad (1)$$

where  $\alpha_{xw}$  is the ratio of the available power at the adaptor input to that at its output, and  $T_a$  is the ambient temperature of the adaptor. Then, in order to determine  $T_x$  by using this relationship,  $\alpha_{xw}$  has to be accurately measured together with  $T_{xw}$  and  $T_a$ . However, it is very hard to determine precisely the magnitude of  $\alpha_{xw}$  through measuring directly its S-parameters and the reflection

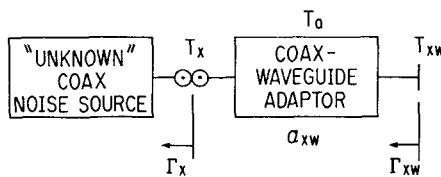


Fig.1 Coaxial noise source whose output port is changed to the waveguide system with a coax-waveguide adaptor.

coefficient of the source to be connected at the input port. On the other hand, a method to avoid the direct measurements of such parameters was proposed by Engen, which can provide the easy measurement of its temperature; i.e. an adaptor-changing method [1].

By extending this idea, we have developed a practical method providing easy determination of its temperature.

### Theory

A schematic diagram depicting the measurement procedure and system is shown in Fig.2 with the symbols representing the relevant parameters. We consider a case where the coaxial connectors are of sexless type such as APC-7 and G-900. The measurement procedure consists of two steps I and II corresponding to the waveguide and the coaxial system, respectively. Each step employs two standard noise sources whose temperatures are different from each other.

First, in the step I, the output port of the unknown source is changed to a waveguide system with a coax-waveguide adaptor and the waveguide

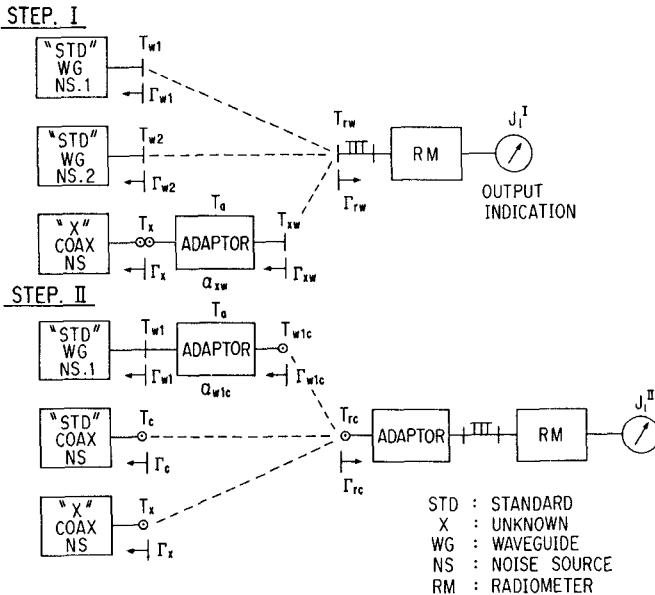


Fig.2 Schematic diagram depicting the measurement procedure and system for a coaxial connector system of sexless type.

unknown is compared with the two standards. Next, in the step II, the adaptor is reversely connected to the waveguide standard to change its output to the coaxial system. And the coaxial unknown is compared with those two standards likewise.

The radiometer output indication is expressed as [2]

$$J_i = G \{ K_{ri} (T_i - T_r) + T_n \} \quad (2)$$

$$K_{ri} = 1 - |\Gamma_{ri}|^2 \quad (3)$$

$$|\Gamma_{ri}| = \sqrt{\frac{\Gamma_i - \Gamma_r^*}{1 - \Gamma_i \Gamma_r}} \quad (4)$$

where  $\Gamma_i$  and  $\Gamma_r$  are the reflection coefficients of a noise source and a radiometer, respectively;  $G$ ,  $T_r$  and  $T_n$  are respectively the conversion coefficient of noise temperature to output indication, the effective temperature when looking into the radiometer at its input port, and an effective temperature due to noises generated within the radiometer. Hereafter, owing to the simplicity, let  $|\Gamma_r| = 0$ , i.e.,  $|\Gamma_{ri}| = |\Gamma_i|$ ,  $K_{ri} = K_i$ .

Here, we introduce the following parameters  $R_i$  and  $R_{II}$  for the respective steps, which are directly measurable from the radiometer indications:

$$R_i = \frac{J_{xw}^I - J_{w1}^I}{J_{w2}^I - J_{w1}^I} \quad (5)$$

$$R_{II} = \frac{J_x^II - J_{w1c}^II}{J_c^II - J_{w1c}^II} \quad (6)$$

For the two noise sources to which the adaptor is connected, the following relationships exist:

$$T_{xw} - T_a = \alpha_{xw} (T_x - T_a) \quad (7)$$

$$T_{w1c} - T_a = \alpha_{w1c} (T_{w1} - T_a) \quad (8)$$

where  $T_a$  is the ambient temperature of the adaptor, and  $\alpha_{xw}$  and  $\alpha_{w1c}$  are available power ratios for the adaptor in the respective steps.

Also we introduce the following relationship for  $\alpha_{xw}$  and  $\alpha_{w1c}$ :

$$\varepsilon = \frac{\alpha_{xw}}{\alpha_{w1c}} - 1 \quad (9)$$

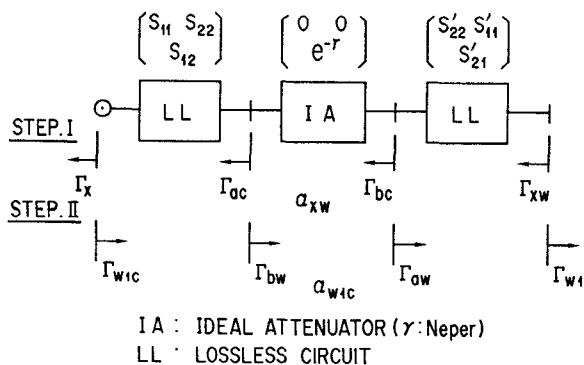


Fig.3 Equivalent circuit of a sexless coax-waveguide adaptor expressed by the modified Wheeler's network.

The maximum and the minimum value of  $\varepsilon$  are approximately given by (see Appendix)

$$\varepsilon_{\max} \cong 4\gamma (|\Gamma_x| + |\Gamma_{w1c}|) \\ \varepsilon_{\min} \cong (|\Gamma_x| - |\Gamma_{w1c}| \pm 2|S_{11}|) \quad (10)$$

The parameters in this expression correspond to the symbols shown in Fig.3 illustrating an equivalent circuit of the adaptor for the steps I and II expressed by the modified Wheeler's network: the  $|S_{11}|$  and  $\gamma$  are can be measured by the sliding short method [3]. The magnitude of  $\varepsilon$  is represented by the mean value from both of the limits as

$$\bar{\varepsilon} \cong 4\gamma (|\Gamma_x|^2 - |\Gamma_{w1c}|^2) \quad (11)$$

Using (2) to (9), we can derive  $T_x$  as follows:

$$T_x = T_a + \frac{-(1+\varepsilon)\{E(T_c-T_a)+F(T_r-T_a)\} + \sqrt{H}}{2(1+\varepsilon)D} \quad (12)$$

where

$$H = (1+\varepsilon)^2 \{E(T_c-T_a)+F(T_r-T_a)\}^2 + 4(1+\varepsilon)D(T_{w1}-T_a) \\ + \{A(T_{w2}-T_a)+B(T_{w1}-T_a)+C(T_r-T_a)\} \quad (13)$$

$$A = \frac{K_{w2}}{K_{xw}} \cdot R_i \quad (14)$$

$$B = \frac{K_{w1}}{K_{xw}} \cdot (1 - R_i) \quad (15)$$

$$C = 1 - A - B \quad (16)$$

$$D = \frac{K_x}{K_{w1c}} \cdot \frac{1}{1 - R_{II}} \quad (17)$$

$$E = -\frac{K_c}{K_{w1c}} \cdot \frac{R_{II}}{1 - R_{II}} \quad (18)$$

$$F = 1 - D - E \quad (19)$$

This equation (12) is complicated but can be simplified, if  $T_{w2}=T_c=T_r=T_a$  and  $|\Gamma_{rw}|=|\Gamma_{rc}|=0$ , as follows:

$$T_x = T_a + \sqrt{\frac{1}{1+\varepsilon} \cdot \frac{K_{w1} K_{w1c}}{K_{xw} K_x} \cdot (1-R_i)(1-R_{II}) \cdot (T_{w1}-T_a)} \quad (20)$$

At the same time,  $\alpha_{xw}$  is given by

$$\alpha_{xw} = (1+\varepsilon) \cdot \frac{K_{w1} K_x}{K_{xw} K_{w1c}} \cdot \frac{1-R_i}{1-R_{II}} \quad (21)$$

Namely, we must use noise sources of the room-temperature type as two standards ( $T_{w1}$  and  $T_c$ ) and a radiometer whose front stage has an isolator at the room temperature.

In a case where the connector system is of sexual type, e.g., the output connector of the unknown is of female type, the measurement procedure consists of three steps as shown in Fig.4: the first step uses a coax(male)-waveguide adaptor for the unknown; the second step uses the same adaptor placed reversely in front of the waveguide standard(NS.1), and at the same time, two coax(male-male) adaptors are placed in front of the coaxial standard and the unknown, respectively; and the third step uses a coax(female-female) adaptor in front of the waveguide standard changed to the coaxial type.

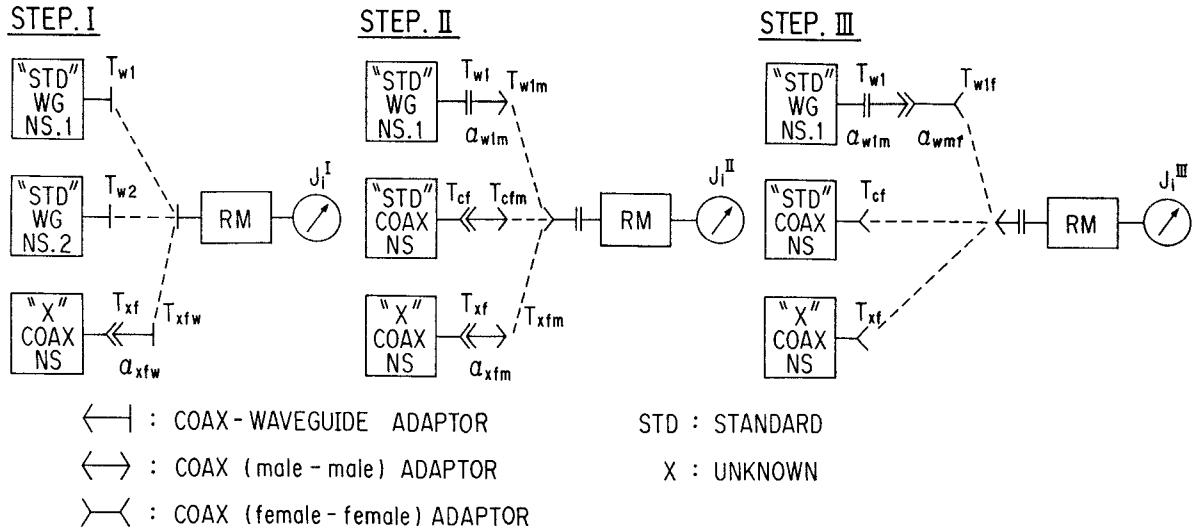


Fig.4 Schematic diagram depicting the measurement procedure and system for a coaxial connector system of sexual type.

Here, we assume that the adaptors of coax(male-male) and (female-female) are almost the same in electrical characteristics. If  $T_w2=Tcfm=Tcf=Tr=Ta$ , and  $|\Gamma_r|=0$ ,  $Txf$  is

$$Txf = Ta + \frac{K_{w1}}{K_{xfw}} \cdot (1-R_I) (T_{w1}-Ta) \cdot \frac{1}{1+\mu} \cdot \sqrt{\frac{K_{w1m} K_{xf}}{K_{xfm} K_{w1f}} \cdot \frac{1-R_{II}}{1-R_{III}} \cdot \frac{1}{1+\nu}} \quad (22)$$

where

$$R_I = \frac{J_{xfw}^I - J_{w1}^I}{J_{w2}^I - J_{w1}^I} \quad (23)$$

$$R_{II} = \frac{J_{xfm}^II - J_{w1m}^II}{J_{cfm}^II - J_{w1m}^II} \quad (24)$$

$$R_{III} = \frac{J_{xf}^III - J_{w1f}^III}{J_{cf}^III - J_{w1f}^III} \quad (25)$$

$$\mu = \frac{\alpha_{xfw}}{\alpha_{w1m}} - 1 \quad (26)$$

$$\nu = \frac{\alpha_{xfm}}{\alpha_{w1m}} - 1 \quad (27)$$

The limits of  $\mu$  and  $\nu$  are respectively given by

$$\mu_{(\max.)} \cong 4\gamma (|\Gamma_{xf}| + |\Gamma_{xfw}|) (|\Gamma_{xf}| - |\Gamma_{xfw}|) \pm 2|S_{11}| \quad (28)$$

$$\nu_{(\max.)} \cong 4\gamma_m \{ (|S_{11}|^2 - |S_{11}|^2) + (|\Gamma_{xf}|^2 - |\Gamma_{w1m}|^2) \pm 2(|S_{11}| |\Gamma_{xf}| + |S_{11}| |\Gamma_{w1m}|) \} + 4|\gamma_m - \gamma_f| |\Gamma_{ef}|^2 \quad (29)$$

The parameters associated to the expressions correspond to the symbols shown in Fig.4 and Fig.5 which illustrates the equivalent circuits of the coaxial adaptors (male-male and female-female) in the second and third steps.

The second term in (29) may be neglected since usually  $|\Gamma_{ef}|^2 \ll 1$  and  $\gamma_m \cong \gamma_f$ .

#### Error Analysis

We consider the major sources of error to temperature calibration by this method when the coaxial connector is of sexless type.

#### 1) Error due to approximation of $\epsilon$ to $\bar{\epsilon}$

The maximum value of the error is

$$|\Delta \epsilon| \cong 8\gamma |S_{11}| (|\Gamma_{xf}| + |\Gamma_{w1m}|) \quad (30)$$

Its contribution to the calibration error is given when  $\epsilon \ll 1$  as

$$|\Delta T_x| \epsilon \cong \frac{1}{2} |T_x - T_a| |\Delta \epsilon| \quad (31)$$

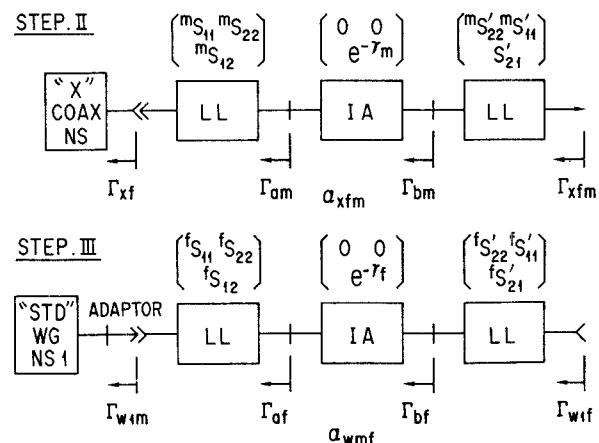


Fig.5 Equivalent circuits of two coaxial adaptors (male-male and female-female) used for the sexual coaxial connector system.

Fig.6 shows an example of the calculation results expressing the relationships of  $|\Delta \epsilon|$  and  $|\Delta T_x|$  to  $\gamma$  in a case where the unknown source is of liquid nitrogen type.

2) Error due to deviation of  $T_w 2$ ,  $T_c$  and  $T_r$  from  $T_a$

Their contributions to the calibration error are

$$|\Delta T_x|_{w2,a} \cong \left| \frac{A(T_{w1}-T_a)}{2D(T_x-T_a)+E(T_c-T_a)} \right| |T_{w2}-T_a| \quad (32)$$

$$|\Delta T_x|_{c,a} \cong \left| \frac{E(T_x-T_a)}{2D(T_x-T_a)+E(T_c-T_a)} \right| |T_c-T_a| \quad (33)$$

$$|\Delta T_x|_{r,a} \cong \left| \frac{C(T_{w1}-T_a)-F(T_x-T_a)}{2D(T_x-T_a)+E(T_c-T_a)} \right| |T_r-T_a| \quad (34)$$

where  $A, C, D, E$  and  $F$  correspond to those in (14) and (16) to (19), respectively, and the magnitudes of  $C$  and  $F$  in (34) are considered much less than unity.

3) Error due to neglection of  $|\Gamma_r|$

When  $|\Gamma_r|=0$ , the maximum difference between  $|\Gamma_{ri}|^2$  and  $|\Gamma_{ri}|^2$  for each of  $K$ 's in (20) is

$$|\Gamma_{ri}|^2 - |\Gamma_{ri}|^2 \cong |\Gamma_r| (1 - |\Gamma_r|^2) |\Gamma_r| - 2 |\Gamma_r| |\Gamma_{ri}| \quad (35)$$

Then,

$$|\Delta T_x|_{ri} \cong \frac{1}{2} (|\Gamma_{ri}|^2 - |\Gamma_{ri}|^2) |T_x - T_a| \quad (36)$$

For example, if  $T_x = 80K$ , and  $|\Gamma_r|=0.01$  and  $|\Gamma_{ri}|=0.1$ , the contribution to the calibration error due to each of  $K$ 's is about 0.21K, so this

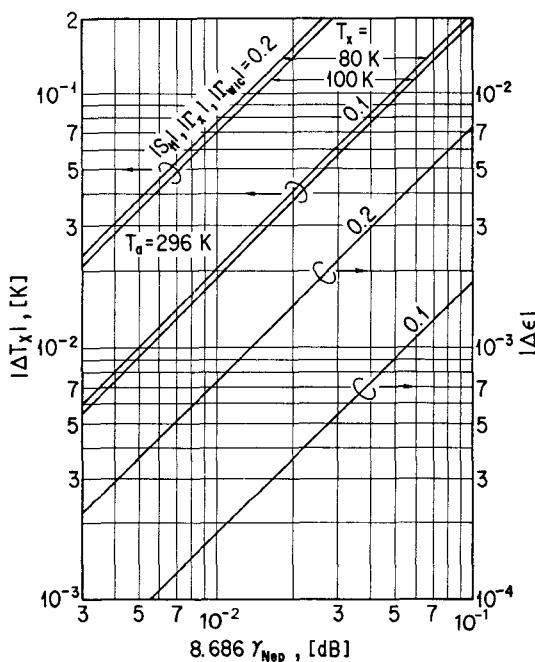


Fig.6 Error due to approximation of  $\epsilon$  to  $\bar{\epsilon}$  and the calibration error of the unknown versus  $\gamma$ .

may become one of significant errors. Therefore, the magnitudes of the reflection coefficients should be as small as possible.

#### 4) Measurement errors of $R_i$ and $R_{ii}$

Their contributions are written as

$$|\Delta T_x|_{Ri} \cong 0.12 |T_x - T_a| |R_i| |\Delta R_i| \quad (37)$$

$$|\Delta T_x|_{Rii} \cong 0.12 |T_x - T_a| |R_{ii}| |\Delta R_{ii}| \quad (38)$$

#### Conclusion

A practical method has been given capable of calibrating easily a coaxial noise source with a waveguide standard without directly evaluating the electrical characteristics of the coax-waveguide adaptor. It has been shown that the measurement equation can be simplified when one of the standards is of room temperature type and the effective temperature of the radiometer when looking inside at the input port is equal to the room temperature, and that the errors due to the deviation of these temperatures from the room temperature can be less influenced as the noise temperature of the waveguide standard is close to that of the unknown.

#### Appendix

$\alpha_{xw}$  and  $\alpha_{w1c}$  shown in Fig.3 are given by

$$\alpha_{xw} = \frac{\varepsilon^{-2Y} (1 - |\Gamma_{ac}|^2)}{1 - \varepsilon^{-4Y} |\Gamma_{ac}|^2} \quad (A.1)$$

$$\alpha_{w1c} = \frac{\varepsilon^{-2Y} (1 - |\Gamma_{aw}|^2)}{1 - \varepsilon^{-4Y} |\Gamma_{aw}|^2} \quad (A.2)$$

where  $\alpha$  is a monotonously decreasing function to  $|\Gamma|$ . By adopting the lossless condition to the LL circuits, these  $|\Gamma_{ac}|$  and  $|\Gamma_{aw}|$  is given by

$$|\Gamma_{ac}| = \left| \frac{S_{11}^* - \Gamma_x}{1 - S_{11} \Gamma_x} \right| \quad (A.3)$$

$$|\Gamma_{aw}| = \left| \frac{S_{11'}^* - \Gamma_{w1}}{1 - S_{11'} \Gamma_{w1}} \right| \quad (A.4)$$

Using these equations and (9), (10) can be derived approximately.

#### References

- [1] G.F.Engen,"A method of calibrating coaxial noise sources in terms of a waveguide standard," IEEE Trans. Microwave Theory Tech., vol.MTT-16, pp.636-639, sept. 1968.
- [2] I.Yokoshima,"Generalized performance analysis of one-port measuring instruments with signal generators," IEEE Trans. Instrum. Meas., vol. IM-21, pp.135-140, May 1972.
- [3] M.Sucher and J.Fox, Handbook of microwave measurements, Third Edition, vol.1. Polytechnic Press of the Polytechnic Institute of Brooklyn.